MET 4501 Spring 2025: In-Class Quiz 5 Name: SOLUTION

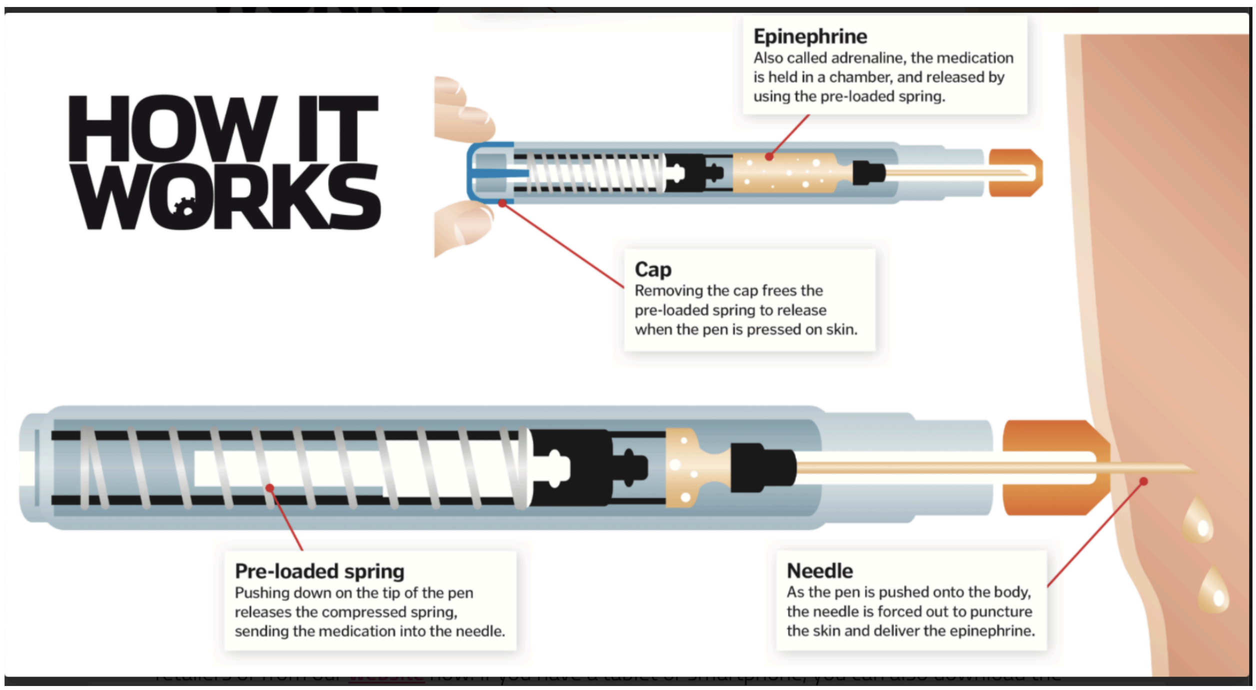
INSTRUCTIONS:

This quiz is open-book, open-note, and you may work with your classmates.

Please answer all questions on your individual papers and submit to me by the end of today's class period.

GIVEN:

A spring inside of an auto-injecting syringe fails catastrophically during use. The spring breaks and ejects violently from the syringe, piercing the user and causing injury. We've been asked to investigate the incident to determine if the failure is due to defective design, inadequate testing or quality control, a material defect, or something else.



The subject helical compression spring, as designed, is made of A313 stainless steel. The outside coil diameter of the spring is measured as 12 mm. The wire diameter is 0.6 mm.

The subject helical compression spring is removed from the auto-injector to measure the free length, which is approximately 70 mm.

The ends are squared and there are 12 ½ total turns. *Note that "closed" and "squared" are synonyms.*

FIND:

If the spring material is yielded during normal use. We'll break that task down into finding the below:

a) The torsional yield strength of the wire, S_{sy}.
b) The static load corresponding to the yield strength, F_y.
c) The spring rate, k.
d) The deflection that would be caused by the load in (b), y_y.
e) The solid length of the spring, L_s.
f) What length should the spring be to ensure that when it is compressed solid and then released, there will be no permanent change in the free length?
g) Comparing this length to the given free length of the subject spring, do we expect the spring material to yield during normal use?

BONUS:
Given the length found in (f), is buckling a possibility? If so, then what additional failure modes should we investigate?

Table 10-5: Mechanical Properties of Some Spring Wires

Material	Elastic Limit, Percent of S_{ut}		Diameter d, in	E		G	
	Tension	Torsion		Mpsi	GPa	Mpsi	GPa
Music wire A228	65 to 75	45 to 60	<0.032	29.5	203.4	12.0	82.7
			0.033 to 0.063	29.0	200	11.85	81.7
			0.064 to 0.125	28.5	196.5	11.75	81.0
			>0.125	28.0	193	11.6	80.0
HD spring A227	60 to 70	45 to 55	<0.032	28.8	198.6	11.7	80.7
			0.033 to 0.063	28.7	197.9	11.6	80.0
			0.064 to 0.125	28.6	197.2	11.5	79.3
			>0.125	28.5	196.5	11.4	78.6
Oil tempered A239	85 to 90	45 to 50		28.5	196.5	11.2	77.2
Valve spring A230	85 to 90	50 to 60		29.5	203.4	11.2	77.2
Chrome-vanadium A231	88 to 93	65 to 75		29.5	203.4	11.2	77.2
A232	88 to 93			29.5	203.4	11.2	77.2
Chrome-silicon A401	85 to 93	65 to 75		29.5	203.4	11.2	77.2
Stainless steel							
A313*	65 to 75	45 to 55		28	193	10	69.0
17-7PH	75 to 80	55 to 60		29.5	208.4	11	75.8
414	65 to 70	42 to 55		29	200	11.2	77.2
420	65 to 75	45 to 55		29	200	11.2	77.2
431	72 to 76	50 to 55		30	206	11.5	79.3
Phosphor-bronze B159	75 to 80	45 to 50		15	103.4	6	41.4
Beryllium-copper B197	70	50		17	117.2	6.5	44.8
Inconel alloy X-750	65 to 70	40 to 45		31	213.7	11.2	77.2

Table 10-1: Formulas for the Dimensional Characteristics of Compression Springs

	Type of Spring Ends			
Term	**Plain**	**Plain and Ground**	**Squared or Closed**	**Squared and Ground**
End coils, N_e	0	1	2	2
Total coils, N_t	N_a	$N_a + 1$	$N_a + 2$	$N_a + 2$
Free length, L_0	$pN_a + d$	$P(N_a + 1)$	$pN_a + 3d$	$pN_a + 2d$
Solid length, L_s	$d(N_t + 1)$	dN_t	$d(N_t + 1)$	dN_t
Pitch, p	$(L_0 - d)/N_a$	$L_0/(N_a + 1)$	$(L_0 - 3d)/N_a$	$(L_0 - 2d)/N_a$

Table 10-4: Constants for Estimating Minimum Tensile Strength of Common Spring Wires

Material	ASTM No.	Exponent m	Diameter, in	A, kpsi · in^m	Diameter, mm	A, Mpa · mm^m	Relative Cost of Wire
Music wire*	A228	0.145	0.004 to 0.256	201	0.10 to 6.5	2211	2.6
OQ&T wire†	A229	0.187	0.020 to 0.500	147	0.5 to 12.7	1855	1.3
Hard-drawn wire‡	A227	0.190	0.028 to 0.500	140	0.7 to 12.7	1783	1.0
Chrome-vanadium wire§	A232	0.168	0.032 to 0.437	169	0.8 to 11.1	2005	3.1
Chrome-silicon wire‖	A401	0.108	0.063 to 0.375	202	1.6 to 9.5	1974	4.0
302 Stainless wire#	A313	0.146	0.013 to 0.10	169	0.3 to 2.5	1867	7.6 to 11
		0.263	0.10 to 0.20	128	2.5 to 5	2065	
		0.478	0.20 to 0.40	90	5 to 10	2911	
Phosphor-bronze wire**	B159	0	0.004 to 0.022	145	0.1 to 0.6	1000	8.0
		0.028	0.022 to 0.075	121	0.6 to 2	913	
		0.064	0.075 to 0.30	110	2 to 7.5	932	

(a) $S_{sy} = 0.45\, S_{ut}$

$$S_{ut} = \frac{A}{d^m} = \frac{1867\ \text{MPa}\cdot\text{mm}^{0.146}}{(0.6\ \text{mm})^{0.146}} = 2011.57\ \text{MPa}$$

$$S_{sy} = 0.45(2011.57\ \text{MPa}) = \boxed{905.2\ \text{MPa}}$$

(b) $\tau = K_B \dfrac{8FD}{\pi d^3} \Rightarrow S_{sy} = K_B \dfrac{8F_y D}{\pi d^3}$

REARRANGING $\Rightarrow F_y = \dfrac{\pi S_{sy} d^3}{8 K_B D}$

$$K_B = \frac{4C+2}{4C-3} = \frac{4(19)+2}{4(19)-3} = 1.068$$

$$C = \frac{D}{d} = \frac{D_o - d}{d} = \frac{12\text{ mm} - 0.6\text{ mm}}{0.6\text{ mm}} = \frac{11.4\text{mm}}{0.6\text{mm}} = 19$$

$$F_y = \frac{\pi (905.2\text{ MPa})(0.6\text{ mm})^3}{8(1.068)(11.4\text{ mm})} = \boxed{6.306\text{ N}}$$

(c) $k \approx \dfrac{d^4 G}{8 D^3 N_a} = \dfrac{(0.6\text{ mm})^4 (69000\text{ MPa})}{8(11.4\text{mm})^3 (10.5)} = \boxed{0.0719 \frac{\text{N}}{\text{mm}}}$

$G = 69.0$ GPa $= 69000$ MPa (TABLE 10-5)

$N_t = N_a + 2 \Rightarrow N_a = N_t - 2 = 12.5 - 2 = 10.5$
(TABLE 10-1)

(d) $F_y = k y_y \Rightarrow y_y = F_y / k$

$$y_y = \frac{6.306\text{ N}}{0.0719\text{ N/mm}} = \boxed{87.76\text{ mm}}$$

(e) $L_s = d(N_t + 1) = (0.6\ mm)(12.5 + 1) = \boxed{8.1\ mm}$

(TABLE 10.1)

(f) $L_{o,max} = L_s + y_y = 8.1\ mm + 87.76\ mm$

$= \boxed{95.86\ mm}$

(g) GIVEN THAT THE MEASURED FREE LENGTH OF THE SPRING IS 70 mm, WHICH IS LESS THAN 95.86 mm, WE DO NOT EXPECT YIELDING OF THIS SPRING DURING FUTURE USE.

(NOTE THAT WE ARE TOLD THE FREE LENGTH WAS MEASURED FROM THE SUBJECT SPRING, WHICH ALREADY EXPERIENCED FAILURE. WE DO NOT KNOW WHAT THE FREE LENGTH OF THE SPRING WAS WHEN IT WAS INITIALLY ASSEMBLED.)

BONUS: TO AVOID BUCKLING,

$$L_0 < \frac{\pi D}{\alpha}\left[\frac{2(E-G)}{2G+E}\right]^{1/2}$$

ASSUME $\alpha = 0.5$ (FIXED ENDS)

$E = 193$ GPa

$$L_0 < \frac{\pi(11.4\text{mm})}{0.5}\left[\frac{2(193-69)}{2(69)+193}\right]^{1/2}$$

$$L_0 < 62.0 \text{ mm}$$

BUCKLING IS A POSSIBILITY.

THE AUTO-INJECTOR HAS A HOUSING AND AN INTERIOR ROD TO SUPPORT THE SPRING & PREVENT BUCKLING. WE SHOULD INVESTIGATE FAILURE MODES OF THESE COMPONENTS NEXT.